## **Chapter 1 Circuit Variables and Elements**

**1.1 Electric Current**

***Definition*** *Current is the rate of flow of electric charge.*

* In an infinitesimal time  the charge  that crosses a given plane xx′ is that contained in the disc of volume  m3, where *A* m2 is the cross-sectional area and *u* m/s is the velocity.



* , where *n* is the concentration of particles per cubic meter and *e* is the positive charge per particle.
*  (1.1.1)
  + 1 A ≡ 1 C/s; dimensions in Equation 1.1.1: ,
* By convention: *the direction of current is that of motion of positive electric charges*.
* From Equation 1.1.1 *i* has the same direction as *u*, when *e* is positive, and is of the opposite direction as *u*, when *e* is negative.

**Current Carriers**

Metals: predominantly *conduction electrons*; in some solids, such as semiconductors, current can also be carried by what are effectively positive charges, or ‘holes’.

Electrolytic solutions: positively charged ions (cations) or negatively charged ions (anions**)**.

Gases: positively charged ions, negatively charged ions, or electrons.

**Example 1.1.1 Drift Velocity in a Copper Wire**

A copper wire in diameter has 8.4 conduction electrons/m3 and carries a current of 10 A. It is required to determine the mean drift velocity of these electrons, assuming the charge per electron is .

***Solution*:** The cross-sectional area *A* of the wire is: 

. Assuming that the current is positive and uniformly distributed across the cross-section of the wire, it follows from Equation 1.1.1 that:  m/s. The negative sign of velocity signifies that the electrons move in a direction opposite that of the current.

**1.2 Voltage**

***Definition*** *The**voltage between two points is the difference in electric potential energy per unit charge, between these points.*



* If  is moved from B to A (Figure 1.2.1), , where  is the work done against the electric field.
* Neglecting second order infinitesimals:

 (1.2.1)

* If a charge of +1 coulomb is moved to a region whose voltage is 1 volt is higher, the increase in electric potential energy is 1 joule.
* Voltage is measured with respect to some arbitrary zero reference, commonly taken as that of the earth.



* + Ground symbol: Figure 1.2.2a;

symbol for voltage reference other than ground: Figure 1.2.2b.

**1.3 Electric Power and Energy**

***Definition*** *Power, p, is the rate at which energy, w, is delivered or absorbed*: *p* = *dw*/*dt*.

*  (1.3.1)
* If  is in amperes and  is in volts,  is in watts (W), or joules/s.
* When the switch is closed, the battery impresses a voltage of 3 V across the metallic filament of the lamp, with terminal a′ at a positive voltage with respect to terminal b′.



* The conduction electrons in the filament have a higher electric potential energy at b′ than at a′. The force of the electric field will therefore drive them down an electric potential energy difference from b′ to a′ through the filament. This loss of electric potential energy is converted to heat in the filament, causing it to glow and emit light.
* In the battery, the chemical reactions of the battery expend energy to move the electrons up an electric potential energy difference from a to b.

*P = VBI* = (3 V)×(0.25 A) = 0.75 W.

**Example 1.3.1 Flashlight Circuit**

If the current is 0.25 A in the circuit of Figure 1.3.1, how many electrons, on average, pass through the lamp per second? What is the electric potential energy of an electron at a relative to that at b? How much energy does the battery deliver in 1 hour?

***Solution*:** A current of +0.25 A in the direction shown corresponds to an electron flow through the lamp of  Since the charge of the electron is  it follows that  electrons/s flow through the lamp, on average.

The electric potential energy difference is the charge times the voltage. Considering the electric potential energy at b to be at zero reference, the electric potential energy of an electron at a is:  Because it is negatively charged, the electron has greater potential energy at b than at a.

Since *P* = 0.75 W, the energy delivered in one hour is  J/hr.

**Example 1.3.2 Discharge of Battery**

An A-size, 1.5 V battery is rated at 3 ampere-hours (Ah). During continuous use, with the battery supplying a current of 100 mA, the battery voltage stays substantially constant at 1.5 V for the first 20 hours. During the next 10 hours, the voltage drops linearly to 1.25 V, while the current drops linearly to 80 mA (Figure 1.3.2). At this point, the battery is no longer considered useful. What was the useful Ah capacity of the battery, and how much energy was delivered by the battery during the 30 hours?



***Solution*:** During the first 20 hours, the battery delivers 0.1×20 = 2 Ah. During the next 10 hours, the current drops linearly with time. The average current during this interval is 90 mA, or 0.09 A. The battery delivers during this interval 0.0910 = 0.9 Ah. The battery therefore delivers 2.9 Ah during the 30 hours of operation.

To determine the energy delivered by the battery, we again have to consider the first 20 hours and the following 10 hours separately. During the first 20 hours, the battery voltage is constant at 1.5 V and the current is constant at 0.1 A. The battery delivers a constant power of 1.50.1 = 0.15 W. The energy delivered up to a time *t* hours is 0.153600*t* = 540*t* J. By the end of this period, the energy delivered is 540×20 = 10,800 J, or 10.8 kJ.

During the interval 20 ≤ *t* ≤ 30 h, *v* = 2 – 0.025*t* V, and *i* =  The instantaneous power *p* is: *p* = *vi* = (2 – 0.025*t*)×(0.14 – 210-3*t*) *=* 0.28 – 0.0075*t* +

510-5*t2* W, where *t* is in hours. The energy delivered is:

1.24 W-hr

To convert to joules, this has to be multiplied by 3,600 s/hr, which gives 4.46 kJ. The total energy delivered by the battery is therefore 10.8 + 4.47 = 15.26 kJ.

**Conservation of Power**

***Concept*** *Conservation of energy implies conservation of power*.

* Energy must be conserved at every instant. It cannot be conserved in a given system over only some finite interval of time, as this would violate conservation of energy.

**1.4 Assigned Positive Directions**

* In circuit analysis, positive directions of unknown voltages or currents are assumed arbitrarily, subject to restrictions imposed by the voltage-current relations of the circuit elements.
* If the *numerical values* of currents and voltages are positive, the actual direction of the given quantity is the same as the assigned positive direction. If the numerical value is negative, the actual direction is opposite that of the assigned positive direction.

***Concept*** *If current i is in the direction of a voltage drop v, then the power p = vi represents power* absorbed *by the element through which i flows*. *Conversely*, *If i is in the direction of a voltage rise v then the power p represents power* delivered *by the element through which i flows*.

* Negative power delivered is power that is actually absorbed, and negative power absorbed is power that is actually delivered.
* *Negative values of current, voltage, or power are perfectly natural in circuit analysis*.

**Example 1.4.1 Assigned Positive Directions of Current and Voltage**

Assume that in the case of the flashlight discussed above, the battery is contained in a box having unmarked terminals, so we do not know which terminal is positive and which is negative. Suppose we decide to assign the positive direction of *VB* as



shown in Figure 1.4.1a. It would then be reasonable to assign the positive direction of  as before, that is, flowing into the lamp at a′.

Suppose that when the current  is measured in the assigned positive direction it is found to be 0.25 A in the direction opposite to that assumed, which means that

*I =* -0.25 A. This would seem to indicate that the positive terminal of the battery is b and its negative terminal is a. We confirm this by measuring the battery voltage  in the assigned positive direction. So . From Figure 1.4.1a, the assigned positive direction of  is in the direction of a voltage drop *VB* in the lamp and a voltage rise *VB* in the battery. Hence, according to Equation 1.3.1, the power absorbed by the lamp, and that delivered by the battery, is as before.

Suppose we had assigned the positive directions of *VB* and  as shown in Figure 1.4.1b, so  is now in the direction of a voltage rise *VB* in the lamp and a voltage drop *VB* in the battery. The same measurements as above, in the assigned positive directions, would give *I* = 0.25 A, and *VB* = -3 V. Equation 1.3.1 now gives:  as the power *delivered* by the lamp and *absorbed* by the battery. The negative sign means that a power of 0.75 W is in fact absorbed by the lamp and delivered by the battery, as before.

It is seen that Equation 1.3.1 gives the same value of power absorbed by the lamp or delivered by the battery, *irrespective of the assigned positive directions of  and * as long as Equation 1.3.1 is interpreted correctly in terms of actual values, whether measured or calculated.

**1.5 Active and Passive Circuit Elements**

***Definition*** *Active devices can generate electrical energy through conversion from another source of energy, whereas passive devices cannot.*

* Active devices, such as batteries and generators, are represented in electric circuits by voltage or current sources. Examples of passive circuit elements are resistors, capacitors, and inductors.

**1.6 Voltage and Current Sources**

***Definition*** *An ideal voltage source maintains a specified voltage across its terminals irrespective of the current through the source.*

* + - Figure 1.6.1a: *v-i* characteristic of an ideal voltage source.



* + *i* through the voltage source depends on the source and on the rest of the circuit.
    - In an **independent voltage source** (Figure 1.6.1b), *vSRC* is specified independently of any other voltage or current in the circuit.
    - In a **dependent voltage source** (Figure 1.6.1c), *vSRC* depends on another voltage or current in the circuit.
* In a **voltage-controlled voltage source** (VCVS), *vSRC* depends on another voltage, e.g., .
* In a **current-controlled voltage source** (CCVS), *vSRC* depends on another current, e.g., .



***Definition*** *An ideal current source maintains a specified current through it irrespective of the voltage across its terminals.*

* + - Figure 1.6.2a: *v-i* characteristic of an ideal current source.
  + *v* across the current

source depends on the source and on the rest of the circuit.

* + - In an **independent current source** (Figure 1.6.2b), *iSRC* is specified independently of any other voltage or current in the circuit.
    - In a **dependent current source** (Figure 1.6.2c), *iSRC* depends on another voltage or current in the circuit.
* In a **voltage-controlled current source** (VCCS), *iSRC* depends on another voltage, e.g., .
* In a **current-controlled current source** (CCCS), *iSRC* depends on another current, e.g., .

**Interconnection of Sources**

***Concept*** *A connection of sources that violates conservation of energy or conservation of charge is invalid and is not allowed in electric circuits.*

**Example 1.6.1 Invalid and Valid Source Connections**

The connection of two ideal voltage sources in parallel (Figure 1.6.3a) is not valid when *VSRC*1 ≠ *VSRC*2, because it violates conservation of energy. If a charge *q* is moved around the circuit, there is a net gain or loss of energy of *q*|*VSRC*1 − *VSRC*2|,unless *VSRC*1 = *VSRC*2.



The connection of two current sources in series as in Figure 1.6.3b is not valid when *ISRC*1 ≠ *ISRC*2, because it violates conservation of charge. If we consider the connection a2b1 between the two sources, then the charge entering this connection per second is *ISRC*2, whereas the charge leaving this connection per second is *ISRC*1. There is a net gain or loss of charge of |*ISRC*1 − *ISRC*2I per second, in violation of conservation of charge, unless *ISRC*1 = *ISRC*2.

Consider an ideal voltage source of  to be connected to an ideal current source of  as in Figure 1.6.4. Such a connection is valid. The current source forces a current of  through the voltage source, and the voltage source impresses a voltage of  across the current source. Since the current through the voltage source is in the direction of a voltage rise across the source, the voltage source *delivers* a power *P* = (12 V)×(2 A) = 24 W. The current through the current source is in the direction of a voltage drop across the source, so this source *absorbs* the same power *P* = (12 V)×(2 A) = 24 W. If the polarity of either source is reversed, the current source delivers power, and the voltage source absorbs power.



**1.7 The Resistor**

**The Nature of Resistance**

***Concept*** *Electrical resistance is due to impediments to the flow of conduction electrons under the influence of an applied electric field.*

* These impediments can be considered to be due to ‘collisions’ between conduction electrons accelerated by the applied electric field and metal atoms of the crystal, which vibrate about their rest positions with an amplitude that increases with temperature.
* As a result of these collisions: i) electrons loose kinetic energy. But because their velocities increase between collisions, the net effect is that they acquire a mean **drift velocity** that is superposed on their much larger random thermal velocities; ii) the metal atoms gain kinetic energy so they vibrate with a larger amplitude. This is reflected as an increase in the temperature of the metal.
* The electrical resistance *R* of a conductor is the ratio of the voltage *v* applied to the conductor to the resulting current *i* in the conductor. Thus:

 (1.7.1)

* When *v* is expressed in volts and *i* in amperes, *R* is in ohms and is denoted by the symbol .
* The reciprocal of the resistance *R* is the **conductance** *G*:

 (1.7.2)

* + When *v* is expressed in volts and *i* in amperes, *G* is in siemens, denoted by the symbol S.

**Ohm’s Law**. For metals at a given temperature, *R* is constant over a wide range of *v* and *i*:

 (1.7.3)

* An ideal resistor obeys Ohm’s law and has a linear *v-i* characteristic whose slope is *R* (Figure 1.7.1a).
* Since such a resistor dissipates power, the assigned positive direction of current *i* is *always* in the direction of the voltage drop *v* across the resistor. With *v* and *i* positive quantities, *R* in Equation 1.7.3 is a positive quantity for a passive resistor.
* A connection of zero resistance, or infinite conductance, is a **short circuit**, whereas a connection of infinite resistance, or zero conductance, is an **open circuit**, since there will be no current flow.



**Power Dissipated**

* Substituting from Equation 1.7.2 or Equation 1.7.3 in Equation 1.1.1 gives for the power absorbed by a resistor:

 (1.7.4)

* + When  is in volts,  is in amperes,  in ohms, and  is in siemens, the power dissipated is in watts.

**Temperature Effect**

* The resistance of a metal increases with temperature, because of the increased amplitude of vibration of metal ions. As a result, the probability of collision increases, the mean time between collisions decreases, the drift velocity and current decrease, and the resistance increases.
* The variation of the resistance of a metal with temperature is given by:

 (1.7.5)

**1.8 The Capacitor**

***Concept*** *The fundamental attribute of a capacitor is its ability to store energy in the electric field resulting from separated positive and negative electric charges.*

* (Figure 1.8.1): example of a capacitor.
*  (1.8.1)
  + If  is in coulombs and  is in volts,  is in farads (F).



* + For the parallel-plate capacitor:

 (1.8.2)

where *ε* is the permittivity.

* In an ideal capacitor,  is constant, so  and  are linearly related, and there is no power dissipation.



***v-i* Relation**

* For the assigned positive directions in Figure 1.8.2, . Substituting in Equation 1.8.1:

, or  (1.8.3)

* *C* must be a positive quantity for a passive capacitor.

**Concept** **Passive Sign Convention**: If the assigned positive direction of current through a passive circuit element is in the direction of a voltage drop across the element, a positive sign is used in the v-i relation of that circuit element. If the assigned positive direction of current through the element is in the

direction of a voltage rise across the element, a negative sign is used in the v-i relation of that circuit element.

* A positive sign is used in the *v-i* relation of a resistor and in Equation 1.8.3 for the assigned positive direction of Figure 1.8.2.
* If  in Figure 1.8.2, *q* increases with time, so current must be flowing into the positive terminal of the capacitor and out of the negative terminal. According to the power flow convention, power is absorbed by the capacitor and stored as electric energy. The values of *i* and  are both positive, so *C* is positive.
* If  *v* > 0, the charge on the capacitor decreases with time, so current must be flowing out of the positive terminal of the capacitor and into the negative terminal. The values of  and  in Equation 1.8.3 are both negative, so  remains a positive quantity. The *actual* current is now in the direction of a voltage rise across the capacitor. This means, according to the power flow convention that the capacitor is now returning to the rest of the circuit energy that had been stored in it.
* If the assigned positive direction of *i* or *v* is reversed in Figure 1.8.2, so that *i* is in the direction of a voltage rise across the capacitor, then Equation 1.8.3 must be written with a negative sign, so that *C* is positive for positive or negative .

**Steady Voltage and Current**

**Concept** When a dc voltage is applied to a capacitor, the resulting current through the capacitor is zero, and the capacitor behaves as an open circuit as far as the dc voltage is concerned.

* For a dc voltage, , and the capacitor charge does not change with time. Hence, *I* = 0, in accordance with Equation 1.8.3, which means that the capacitor behaves as an open circuit as far as the dc voltage is concerned.

**Stored Energy**

* The energy stored in the electric field of a capacitor is equal to work done in separating the charges  and.
* From Equation 1.2.1,  For a capacitor,  and  Substituting in the expression for 

 (1.8.4)

**1.9 The Inductor**

***Concept*** *The fundamental attribute of an inductor is its ability to store energy in the magnetic field produced by a current.*

* A prototypical inductor is a coil of *N* turns that is wound on a toroidal core of magnetic material (Figure 1.9.1).



* Because of the toroidal shape, and if the winding is tight, with negligible thickness of the wire and insulation, the flux is confined to the core and links all the turns of the coil, the flux linkage *λ* being equal to *Nφ*.
* The inductance *L* of the coil is defined as the flux linkage per unit current of the coil, or:

*λ* = *Li* (1.9.1)

* When *i* is in amperes and *λ* is in weber-turns (Wb-turns), *L* is in henries (H).
  + In an ideal inductor, *L* is constant, so *λ* and *i* are linearly related, and there is no power dissipation.

***v-i* Relation**

* If the flux changes with time, a voltage is induced in the coil whose magnitude is given by Faraday’s law:

 (1.9.2)

* Substituting from Equation 1.9.1:

, or  (1.9.3)

* *L* must be a positive quantity for a passive inductor.
* According to Lenz’s law, the polarity of *v* due to a change in *i* is such that it opposes the change in *i*. If  *v* opposes the increase in *i* by being a voltage drop in the direction of *i* (Figure 1.9.2). Power is absorbed by the inductor and stored as energy in the magnetic field; Equation 1.9.3 are written with a positive sign.



* If  *v* opposes the decrease in *i* by being a voltage rise in the direction of  thereby adding to *vSRC* and aiding the flow of current. The inductor now returns to the rest of the circuit, energy that had previously been stored. In both cases, *L* is a positive quantity in Equation 1.9.3.
* The passive sign convention applies, so that if the assigned positive direction of *i* is in the direction of a voltage rise across the inductor, Equation 1.9.3 are written with a negative sign, and *L* remains a positive quantity.

**Example 1.9.1 Inductance of Toroidal Coil**

Consider the toroidal core shown in Figure 1.9.1. Let the radius of the toroid be *a* m, the cross-sectional area of the core be *A* m2, and its relative permeability be *μr*. The cross-sectional dimensions of the core are assumed to be small compared to *a*, so that the magnetic field intensity *H* can be considered constant across the cross section of the core. A coil of *N* turns is tightly wound around the core so that all the magnetic flux lies entirely in the core. It is required to determine the inductance of the toroidal coil.

***Solution*:** Consider a circle of radius *r* that lies inside the core. From ampere’s circuital law: *H*×2*πr = NI*, where *I* is the current in the coil, so that:

 (1.9.4)

The magnetic flux density *B* is:

 (1.9.5)

where *μ*0 is the permeability of free space (4π×10−7 henries/meter). Because the dimensions of the core are small compared to that of the toroid, we may replace  with  so that the flux in the core is:

 (1.9.6)

The flux linkage is:

 (1.9.7)

It follows from Equation 1.9.1 that the inductance of the coil is:

 (1.9.8)

**Steady Voltage and Current**

**Concept** When a dc current flows through an inductor, the voltage across the inductor is zero, and the inductor behaves as a short circuit as far as the dc current is concerned.

* When the current is dc, , and the flux does not change with time. Hence, *V* = 0, which means that the inductor behaves as a short circuit.

**Stored Energy**

* If the current  in the inductor, and hence the flux linkage  are positive and increasing with time, the power input to the inductor is:  The work done in increasing  from 0 at  to at time ** which equals the energy stored in the magnetic field, is:

 (1.9.9)

**1.10 Concluding Remarks**

* Fundamentally, the three basic circuit parameters of resistance, capacitance, and inductance model three essential attributes of the electromagnetic field associated with voltages and currents. These attributes are: power dissipation, energy stored in the electric field, and energy stored in the magnetic field.
* Electromagnetic fields are inherently distributed and travel in any electrical system as waves at speeds that may approach that of light in vacuum. The wavelength equals the speed of the wave divided by the frequency of the voltage and current signals.
* As long as the physical dimensions of the system are small compared to the wavelength, the wave nature of electric signals can be neglected. This is tantamount to ignoring the distributed nature of the power dissipation, the energy stored in the electric field, and that stored in the magnetic field. Each of these entities is then separately lumped and represented by discrete circuit elements, namely, resistors, capacitors, and inductors.
* In basic circuit theory it is assumed that the circuit parameters – *R*, *C*, and *L* – are constant, that is, they do not vary with current or voltage, nor with time, which makes the corresponding *v-i*, *v-q*, or *λ-i* relations linear. The system is designated as linear, time-invariant (LTI). A linear system obeys superposition.
* Combinations of basic circuit elements are extremely useful not just for modeling but also for **signal processing**, that is, modifying, in a prescribed manner, the magnitude and time course of current or voltage signals.